A flexible two-step randomised response model for estimating the proportions of individuals with sensitive attributes

Anne-Françoise Donneau, Murielle Mauer
Francisco Sartor and Adelin Albert

Department of Biostatistics, University of Liège, Liège
Scientific Institute of Public Health –Louis Pasteur, Brussels

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Problem definition

Let p be the proportion of individuals in a given population presenting a certain characteristic of interest (e.g., smoking, heart defect, diabetes,…).

Problem: Estimating p from a sample of size n drawn from the population

Classical studies: Let r be the number of subjects with the characteristic in the sample, then an estimate of p is given by

$$\hat{p} = \frac{r}{n} \quad \text{and} \quad SE(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$IC\ \ 95\%: \hat{p} - 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq p \leq \hat{p} + 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Difficulties

When the attribute of interest is a “sensitive” one (e.g., illicit drugs consumption, psychiatric disorder,..), people tend to refuse to answer or intentionally mislead

↓

biased estimation of the true proportion p in classical method

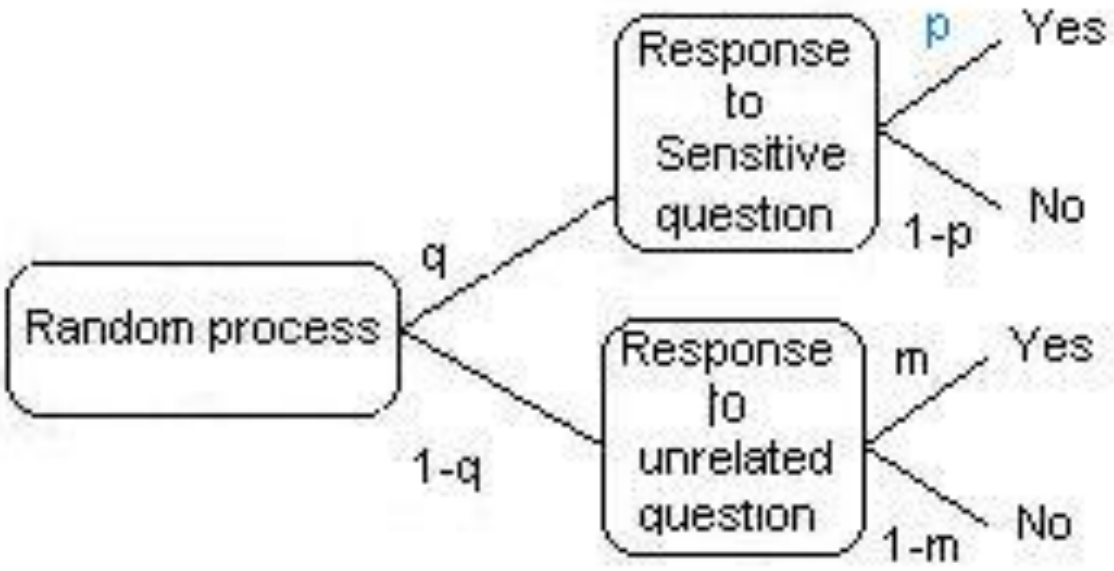
Solution to reduce or eliminate bias linked to the wish to keep private life secret →Random Response Model (RRM) (Warner, 1965)

RRM approach (I)

- Objective : reassuring the respondent that a possible affirmative answer to sensitive questions can't put him/her in a dangerous situation

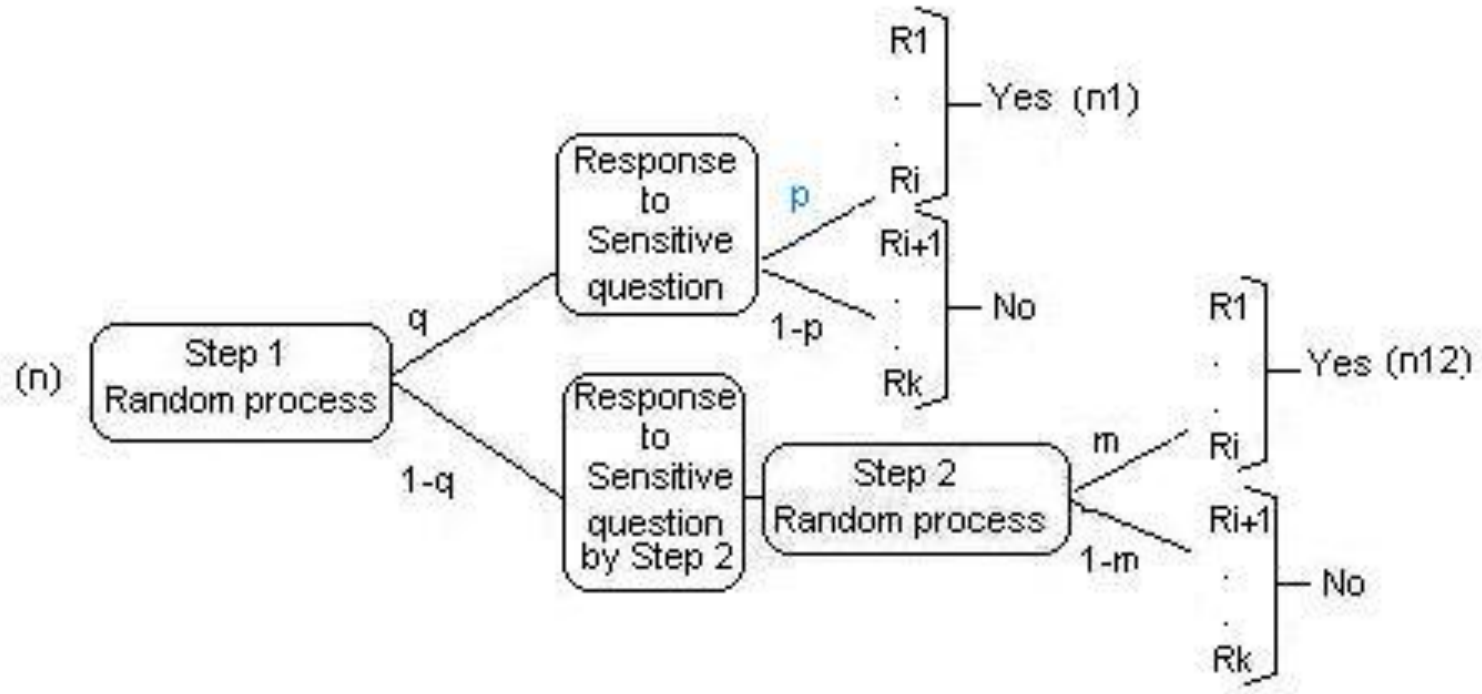
- Method: - Pair the sensitive question with an unrelated one.
  - Respondent select randomly one of these two questions.
  - Interviewer records only a "Yes or No" without knowing which question has been answered.

RRM approach (II)



New approach (I)

- No unrelated question but merely the sensitive question.
- Question with categorical answers which can be combined with some flexibility to have a binary outcome.



New approach (II)

Let f denote the probability of a positive answer regardless of the question,

$$f = qp + m(1-q)$$

Then, as the probabilities of the 2 random processes q and m are supposed to be known (and can be fixed a priori by the investigator), the probability of a positive answer to the sensitive question is

$$p = \frac{f - m(1-q)}{q}$$

Properties (I)

If n_1 and n_{12} are the observed numbers of subjects responding positively according to the outcome of the Step1 and Step1+ Step2, respectively

$$\hat{f} = \frac{n_1 + n_{12}}{n}$$

Consequently, $\hat{p} = \frac{\hat{f} - m(1-q)}{q}$ if the right hand-side is >0

and $\hat{p} = 0$ otherwise;

Properties (II)

Statement1: $\hat{p}$ is an unbiased and consistent estimator of p

- Unbiasedness $E(\hat{p}) = E\left[\dfrac{\hat{f} - m(1-q)}{q}\right]$

$$= \frac{E\left(\hat{f}\right) - m(1-q)}{q}$$

As $\hat{f}$ is an unbiased estimator of f

$$\rightarrow E(\hat{p}) = \frac{f - m(1-q)}{q} = p$$

- Consistency

By the law of large sample, $\hat{f} \rightarrow f$

Hence,

$$\hat{p} = \frac{\hat{f} - m(1-q)}{q} \rightarrow p$$

Properties (III)

Statment2 : $Var(\hat{p}) = \frac{1}{nq^2} f(1-f)$

$$Var(\hat{p}) = \left(\frac{\hat{f} - m(1-q)}{q} \right)$$

$$= \frac{1}{q^2} Var(\hat{f})$$

Since m and q are constant and as $Var(\hat{f}) = \frac{f(1-f)}{n}$

$$Var(\hat{p}) = \frac{1}{nq^2} f(1-f)$$

Properties (VI)

Thus for large n,

$$\frac{\hat{p}-p}{\sqrt{\frac{1}{nq^2}\hat{f}(1-\hat{f})}} \sim N(0,1)$$

Confidence interval

$$\left[\hat{p}-Q_Z(1-\frac{\alpha}{2})\sqrt{\frac{1}{nq^2}\hat{f}(1-\hat{f})};\ \hat{p}+Q_Z(1-\frac{\alpha}{2})\sqrt{\frac{1}{nq^2}\hat{f}(1-\hat{f})}\right]$$

One-sample power (I)

Determination of sample size n, so that $\left|\hat{p}-p\right| \le \Delta$ at the α% significance level

$$\Delta = Q_{z(1-\frac{\alpha}{2})}\sqrt{\frac{1}{nq^2}f(1-f)} \rightarrow n = \frac{Q_z^{2(1-\frac{\alpha}{2})}f(1-f)}{q^2\Delta}$$

As $f = qp + m(1-q)$,

$$n = \frac{Q_z^{2(1-\frac{\alpha}{2})}\left[qp+m(1-q)\right]\left[1-qp-m(1-q)\right]}{q^2\Delta}$$

One-sample problem (II)

Example based on the prevalence of the use of cannabis.

Hypothesis: $\Delta = 2\%$

$\alpha = 5\%$

$m = 5/6$ and $q = 0.5$

$p = 25\%$

Plugging these values in the previous formula, provides

$$\rightarrow \quad n \approx 191$$

Application : Question

During all life, how many times have you used illicit drug ?

1 = Never
2 = 1-2 times
3 = 3-5 times
4 = 6-9 times
5 = 10-19 times
6 = 20 times or more

better than

0=No
1=Yes

Application: Random process

Flip the coin (RP1), if the result is tail answer to question « a ». If the result is head, go to « b ».

a) During all life, how many times have you used illicit drug ?

1 = Never
2 = 1-2 times
3 = 3-5 times
4 = 6-9 times
5 = 10-19 times
6 = 20 times or more

Response ☐

b) Roll the dice once (RP2). What is the result ?

Application: Study material

Undergraduate students registered at the University of Liège during the academic year 2003-2004

Characteristics of the students (n=435)		
Age (year)		18.1 ± 0.78
Gender	Female (%)	288 (66.4)
	Male(%)	146 (33.6)

Application: Practical aspect

- During supervised practical works

  → more receptive and concentrated

- Explanation

- Closed box

Application: Results

Sensitive Question

"Yes" answer : 2-6 (1 or more)

"No" answer : 1 (Never)

$\longrightarrow$ m = 5/6

Sensitive question	Prevalence (± SE)	95% CI
Illicit drug	40.9 (± 4.7)	31.7 - 50.0

Application: Flexibility (I)

Sensitive Question

“Yes” answer : 4-6 (6 or more)

“No” answer : 1-3 (Never - 5 times)

$\rightarrow$ m = 1/2

Sensitive question	Prevalence (± SE)	95% CI
Illicit drug	26.7 (± 4.7)	17.6 – 35.9

Application: Flexibility (II)

Sensitive Question

“Yes” answer : 6 (20 or more)

“No” answer : 1-5 (Never - 19 times)

$\rightarrow$ m = 1/6

Sensitive question	Prevalence (± SE)	95% CI
Illicit drug	16.4 (± 3.6)	9.3 – 23.8

Application: Comparison anonymous questionnaire

A classical anonymous questionnaire was distributed to n = 462 undergraduate students also registered at the University of Liège during the academic year 2003-2004 .

Illicit drug	Prevalence (± SE)
Classical (n=462)	24.2 (± 2.0)
RRM (n=435)	16.4 (± 3.6)

Conclusion

- The problem of estimating the prevalence of sensitive attributes is quite common (public health, social life, economy,…)
- New approach of RRM by a two-step (RP1 – RP2) rather than by a one-step (RP1) random process response
- Flexibility with RP2 (question with multiple answers, modify)
- Can the two-step RRM be a substitute for the classical approach in general?
- Sensitivity analyses should be carried on .